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in *l'Intermediaire de Mathematiciens* as Question 3667, suggested to Prof. P. Barbarin (Paris) a more general problem, viz., *Mener par un point donné dans un angle une secante de longueur donnés,* of which a complete solution has been published by himself in *l'Enseignement Mathematique* (XIII^e, 1, 1911, pp. 15-23).

The investigation is carried out analytically, and the following conclusions have been drawn. The generalized problem is solved algebraically by an equation of the third or fourth degree, and graphically by the intersection of a circle with an hyperbola. Special cases have been shown where the general equation can be lowered to second degree (where the Euclidean construction is possible). There is also a particular case, where the problem is reduced to that of the tri-section of an angle.

A solution of this problem as No. 364 is given by C. N. Schmall on page 140-141, Vol. XVII of the MONTHLY.

MECHANICS.

253. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Stroud, England.

R_1 and R_2 are ranges on a horizontal plane of particles projected with given velocity from A on the plane to pass through B. Show that $a(R_1+R_2)-R_1R_2=\frac{a^4}{c^2}$, where $c=AB$ and a is the horizontal projection of AB.

I. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Let h be the distance of B above the horizontal plane, v the velocity of projection, α and β the two angles of projection which make the particles pass through B. Let $2v^2/g=m$.

$$R_1=\frac{m}{2}\sin 2\alpha=\frac{m\tan\alpha}{1+\tan^2\alpha},\quad R_2=\frac{m\tan\beta}{1+\tan^2\beta}.$$

$$R_1+R_2=\frac{m(\tan\alpha+\tan\beta)(1+\tan\alpha\tan\beta)}{(1+\tan^2\alpha)(1+\tan^2\beta)},\quad R_1R_2=\frac{m^2\tan\alpha\tan\beta}{(1+\tan^2\alpha)(1+\tan^2\beta)}.$$

The equation of the trajectory is

$$y=x\tan\alpha-\frac{x^2}{m}\sec^2\alpha=x\tan\alpha-\frac{x^2}{m}(1+\tan^2\alpha).$$

Since B lies on this, we have $h=a\tan\alpha-\frac{a^2}{m}-\frac{a^2\tan^2\alpha}{m}$.

Solving as a quadratic in tan α we have

$$\tan\alpha = \frac{m+\sqrt{(m^2-4hm-4a^2)}}{2a}.$$

Similarly,

$$\tan\beta = \frac{m-\sqrt{(m^2-4hm-4a^2)}}{2a}.$$

Whence $\tan\alpha+\tan\beta = \frac{m}{a}$, $\tan\alpha\tan\beta = \frac{(h\,m+a^2)}{a^2}$,

$$\tan^2\alpha+\tan^2\beta = \frac{(m^2-2hm-2a^2)}{a^2},$$

$$(1+\tan^2\alpha)(1+\tan^2\beta) = \frac{m^2}{a^4}(a^2+h^2) = \frac{m^2c^2}{a^4}.$$

$$a(R_1+R_2)-R_1R_2 = \frac{a^4}{m^2c^2}.m^2[1+\tan\alpha\tan\beta - \tan\alpha\tan\beta] = \frac{a^4}{c^2}.$$

II. Solution by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

Let α=the angle of projection of one particle, and β the angle of the other; R_1, the range of the former, and R_2, the range of the latter; and v, the velocity of the particles. The equations of the paths of the two particles are

$$y = x\tan\alpha - \frac{g\,x^2}{2v^2\cos^2\alpha}, \text{ and } y = x\tan\beta - \frac{g\,x^2}{2v^2\cos^2\beta}$$

and the equation of the inclined plane is $y = x\tan\gamma = x.\frac{\sqrt{(c^2-a^2)}}{a}$. Now it is easily shown that

$$R_1 = \frac{v^2\sin 2\alpha}{g} \ldots(1), \text{ and } R_2 = \frac{v^2\sin 2\beta}{g} \ldots(2).$$

Solving the equation of the plane with each of the equations of the paths of the particles, we easily find that

$$\cos^2\alpha = \frac{ga(R_1-a)}{2v^2\sqrt{(c^2-a^2)}} \ldots(3), \text{ and } \cos^2\beta = \frac{ga(R_2-a)}{2v^2\sqrt{(c^2-a^2)}} \ldots(4).$$

From (1), $\sin 2\alpha = \frac{gR_1}{v^2}$, or $\sin\alpha\cos\alpha = \frac{gR_1}{2v^2} \ldots(5)$, and from (2),

$$\sin\beta\cos\beta=\frac{gR_2}{2v^2}\ldots(6).$$

Squaring (5), and dividing by (3), we have

$$\sin^2\alpha=\frac{gR_1^2\sqrt{(c^2-a^2)}}{2av^2(R_1-a)}\ldots(7).$$

Squaring (6) and dividing the result by (4), we have,

$$\sin^2\beta=\frac{gR_2^2\sqrt{(c^2-a^2)}}{4av^2(R_2-a)}\ldots(8).$$

Equating the sum of (3) and (7), with the sum of (4) and (8), we have, after cancelling out common factors,

$$\frac{R_1^2\sqrt{(c^2-a^2)}}{2a(R_1-a)}+\frac{a(R_1-a)}{2\sqrt{(c^2-a^2)}}=\frac{R_2^2\sqrt{(c^2-a^2)}}{2a(R_1-a)}+\frac{a(R_2-a)}{2\sqrt{(c^2-a^2)}}.$$

Whence, $\frac{R_1^2(c^2-a^2)}{R_1-a}+a^2(R_1-a)=\frac{R_2^2(c^2-a^2)}{R_2-a}+a^2(R_2-a)$, or

$$\left[\frac{R_1^2}{R_1-a}-\frac{R_2^2}{R_2-a}\right](c^2-a^2)=a^2(R_2-R_1),$$

$$[R_1^2(R_2-a)-R_2^2(R_1-a)](c^2-a^2)=a^2(R_2-R_1)(R_2-a)(R_1-a),$$

$$[R_2R_2(R_1-R_2)-a(R_1^2-R_2^2)](c^2-a^2)=a^2(R_2-R_1)(R_2-a)(R_1-a).$$

$$\therefore [a(R_1+R_2)-R_1R_2](c^2-a^2)=a^2[R_1R_2-a(R_1+R_2)+a^2].$$

$$\therefore [a(R_1+R_2)-R_1R_2=a^4/c^2.$$

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

Edited by Dr. G. E. Wahlin, University of Illinois.

178. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Find a formula which gives all the integral solutions prime to 5 of the congruence $x^2+y^2\equiv0 \pmod{5^4}$.